Physica E 41 (2009) 718–722

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Maximum entangled states and quantum teleportation via single Cooper pair box

N. Metwally [a,b,*], A.A. El-Amin [c]

[a] Mathematics Department, College of Science, Bahrain University, 32038, Bahrain
[b] Mathematics Department, Faculty of Science, South Valley University, Aswan, Egypt
[c] Physics Department, Faculty of Science, South Valley University, Aswan, Egypt

ARTICLE INFO

Article history:
Received 4 September 2008
Received in revised form
13 November 2008
Accepted 27 November 2008
Available online 9 December 2008

PACS:
03.65.Ud
03.67−a
03.67.HK

Keywords:
Charge qubit
Entanglement and teleportation

ABSTRACT

We study a single Cooper pair box interacting with a single cavity mode and discuss the roles played by the detuning parameters and charged capacities on the degree of entanglement. For large values of the detuning parameter the survival of the entanglement is increased. We generate a maximum entangled state and use it to perform the original teleportation protocol. The fidelity of the teleportated state is increased due to decrease in the detuning parameter and the number of photon inside the cavity.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

Several schemes have been proposed for implementing quantum computer hardware in solid state quantum electronics. These schemes use electric charge [1], magnetic flux [2], superconducting phase [3] and electron spin [4]. The basic element of the quantum information is the quantum bit (qubit), which is considered as a two level system. Consequently, most of the researches concentrate to generate entanglement between two level systems [5]. Among these systems, the Cooper charged pairs, due to its properties as a two-level quantum system, which makes it a qubit candidate in a quantum computer [6,7]. One of the most important tasks in the context of quantum information is the quantum teleportation, which emerges from the quantum entanglement. Quantum teleportation protocol was first introduced by Bennett et al. [8], and has been investigated by many authors [9–11].

In our contribution, we have considered a system consists of a single Cooper pair box interacting with a cavity mode. The separability problem is investigated, where the intervals of time in which the generated state is entangled or separable are determined. On the other hand, under a specific circumstance one can use this system to generate a maximum entangled state. Finally we use the generated entangled state to perform the quantum teleportation.

The paper is organized as follows: The description of the system and its solution are introduced in Section 2. In Section 3, the separability problem and the degree of entanglement contained in the generated entangled state are investigated. Section 4, is devoted to study the effect of the field and the charged qubit quantum teleportation.

2. The model and its evolution

The single superconducting charged qubit consists of a small superconducting island with Cooper pair charge Q. This island connected by two identical Josephson junctions, with capacitance C_j and Josephson coupling energy E_j, to a superconducting electrode [11–14]. This system is described by the Hamiltonian,

$$H_s = 4E_c(n - n_g)^2 - E_j \cos\phi, \tag{1}$$

where $E_c = e^2/2(C_g + C_j)$ is the charging energy, $E_j = \hbar I_c/2e$ is the Josephson coupling energy, e is the charge of the electron, $n_g = C_g V_g/2e$ is the dimensionless gate charge, C_g is the gate capacitance, v_g is controllable gate voltage, n is number operator of excess Cooper pair on the island and ϕ is phase operator [12].

* Corresponding author.
E-mail address: NMetwally@gmail.com (N. Metwally).

1386-9477/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.physe.2008.11.012

The Hamiltonian of system (1) can be simplified to a very simple form, if the Josephson coupling energy E_j is much smaller than the charging energy i.e. $E_j \ll E_c$. In this case, the Hamiltonian of the system can be parameterized by the number of Cooper pairs n on the island. If the temperature is low enough, the system can be reduced to two-state system (qubit) [16,15],

$$H_s = -\tfrac{1}{2}B_z\sigma_z - \tfrac{1}{2}B_x\sigma_x, \tag{2}$$

where $B_z = E_{cl}(1-2n_j)$, E_{cl} is the electric energy and $B_x = E_j$ and σ_z, σ_z are Pauli matrices. This Cooper pair can be viewed as an atom with large dipole moment coupled to microwave frequency photons in a quasi-one-dimensional transmission line cavity (a coplanar waveguide resonator). The combined Hamiltonian for qubit and transmission line cavity is given by

$$H = \omega a^\dagger a + \omega_c\sigma_z - \lambda(\mu - \cos\theta\sigma_z + \sin\theta\sigma_x)(a^\dagger + a), \tag{3}$$

where ω is the cavity resonance frequency, $\omega_c = \sqrt{E_j^2 + [4E_c(1-2n_g)]^2}$ is the transition frequency of the Cooper pair qubit, $\lambda = (\sqrt{C_j}/(C_g + C_j))\sqrt{e^2\omega/2\hbar}$, is coupling strength of resonator to the Cooper pair qubit, $\mu = 1 - n_g$ and $\theta = \tan^{-1}\{E_j/E_c(1-2n_g)\}$ is the mixing angle. The density operator according to the hamiltonian (3) [16] is

$$\begin{aligned}\rho(t) = {} & A(|\alpha|^2|g,n\rangle\langle g,n| + |\beta|^2|e,n\rangle\langle e,n|)\\ & + B\alpha\beta^*|g,n\rangle\langle e,n| + B^*\alpha^*\beta|e,n\rangle\langle g,n|\\ & + \mathrm{i}C\sqrt{n+1}(\alpha\beta^*|g,n\rangle\langle g,n+1| + \beta\alpha^*|g,n+1\rangle\langle g,n|)\\ & - \mathrm{i}C(\sqrt{n+1}|\beta|^2|e,n\rangle\langle g,n-1|\\ & + \sqrt{n}\beta\alpha^*|e,n\rangle\langle e,n-1|)\\ & + \mathrm{i}\eta C^*(\sqrt{n+1}|\beta|^2|g,n+1\rangle\langle e,n|\\ & - \sqrt{n}|\alpha|^2|g,n\rangle\langle e,n-1|)\\ & + \eta^2 S_n^2\sqrt{n}\sqrt{n+1}(\beta\alpha^*|g,n+1\rangle\langle e,n-1|\\ & + \alpha\beta^*|e,n-1\rangle\langle g,n+1|)\\ & + S_n^2(n|\alpha|^2|e,n-1\rangle\langle e,n-1|\\ & + (n+1)|\beta|^2|g,n+1\rangle\langle g,n+1|)\\ & + \mathrm{i}\sqrt{n}S_n(|\alpha|^2C|e,n-1\rangle\langle g,n|\\ & + \alpha\beta^*C^*|e,n-1\rangle\langle e,n|),\end{aligned} \tag{4}$$

where

$$\begin{aligned}&A = C_{n+1}^2 + \frac{\Delta^2}{4}S_{n+1}^2, \quad B = C_{n+1}^2 - \frac{\Delta^2}{4}S_{n+1}^2 - \mathrm{i}\Delta S_{n+1}C_{n+1}\\ &C = \mathrm{i}S_n\left(C_{n+1} + \frac{\mathrm{i}\Delta}{2}S_{n+1}\right), \quad C_n = \cos(\gamma\mu_n\tau),\\ &S_n = \frac{\sin(\gamma\mu_n\tau)}{\mu_n}, \quad \gamma = \frac{\sqrt{C_j}}{C_g + C_j},\\ &\tau = \sqrt{\frac{e^2\omega}{2\hbar}}t \ \text{ is the scaled time}\end{aligned} \tag{5}$$

with $\mu_n = \sqrt{\Delta^2/4 + \lambda n}$, $\Delta = E_j - \omega$ is the detuning between the Josephson energy and the cavity field frequency.

We may refer here to the work given in Refs. [17,18] where the present system has been discussed with its dissipative extension where the asymptotic dynamics of a driven spin-boson system with the environment formed by a broadened localized mode. Also, the spin-boson system is analyzed numerically and analytically. The engineering of entanglement has been discussed for the single Cooper pair box with phase damping [19].

3. Degree of entanglement

In this section, we study the behavior of the output state (4) from the separability point of view. To achieve this task, we plot the time evolution of the eigenvalues of the partial transpose eigenvalues of the output density operator ρ^{T_2} [20,21]. Also we investigate the time development of the occupation probabilities and the degree of entanglement in the output state.

There are different measures known for quantifying the degree of entanglement in a bipartite system, such as the entanglement of formation [22,23], entanglement of distillation [24] and negativity [26]. In our calculations we consider the concurrence as a measure of the degree of entanglement. For two qubits, the concurrence is calculated in terms of the eigenvalues η_1, η_2, η_3 and η_4 of the matrix $R = \rho\sigma_y \otimes \sigma_y\rho^*\sigma_y \otimes \sigma_y$. It is given by

$$\begin{aligned}&\mathscr{C} = \max\{0, \sqrt{\eta_1} - \sqrt{\eta_2} - \sqrt{\eta_3} - \sqrt{\eta_4}\}\\ &\text{where } \eta_1 \geqslant \eta_2 \geqslant \eta_3 \geqslant \eta_4.\end{aligned} \tag{6}$$

For maximally entangled states $\mathscr{C} = 1$ while $\mathscr{C} = 0$ for separable states [22,24]. In this context, we indicate that concurrence (6) is related to the eigenvalues of the partial transpose of $\rho(t)$ by

$$\mathscr{C} = 2\max\{0, |\kappa_n|\}, \tag{7}$$

where κ_n is the negative eigenvalues of $\rho^{T_2}(t)$ [25]. In the first example, we assume that the Cooper pair box is initially prepared in the ground state and the field is prepared in the Fock state, i.e. the initial state of the system is given by $|g,n\rangle$. In Fig. 1, we investigate the effect of the detuning parameter on the behavior of the eigenvalues of the partial transpose of the density operator of the output state as well as the time evolution of the occupation probabilities and the degree of entanglement. We assume that the ratio, $C_{jg} = C_j/C_g$ between the Josephson junction capacity, C_j, and the gate capacity C_g, is defined by $C_j/C_g = \frac{5}{2}$ and $n = 1$. In Fig. (1a), we see that as the interaction goes on, i.e. the scaled time, $\tau > 0$, there is only one negative eigenvalue and the rest are non-negative. According to the Peres–Horodecki criterion [20,21], there is a generated entangled state. For larger time $\tau \simeq 5$ all the eigenvalues are non-negative. This means that the entangled qubit turns into a product state (separable). In Fig. 1c, we plot the occupation probabilities against the scaled time. Note that the populations of the four states have different non-zero values for the diagonal occupations whilst the off-diagonal elements are identical. For this reason an entangled state is generated as soon as the interaction starts. In this case the entangled state is

$$\begin{aligned}\rho = {} & \kappa(|n,g\rangle\langle n-1,e| + |n-1,e\rangle\langle n,g|) + \chi_1|n,g\rangle\langle n,g|\\ & + \chi_2|n-1,e\rangle\langle n-1,e|,\end{aligned} \tag{8}$$

where χ_1 and χ_2 are the probabilities to find the system in the states $|n,g\rangle\langle n,g|$ and $|n-1,e\rangle\langle n-1,e|$, respectively. In computational basis one can write the density operator (8) as $\rho = \kappa(|1,0\rangle\langle 0,1| + |0,1\rangle\langle 1,0|) + \chi_1|1,0\rangle\langle 1,0| + \chi_2|0,1\rangle\langle 0,1|$. It is clear that $|n,g\rangle\langle n-1,e| + |n-1,e\rangle\langle n,g|$ is one type of Bell state, $|\psi^+\rangle\langle\psi^+|$. However at $\tau \simeq 2.5$, the partially entangled state (8) becomes a maximum, where all the occupation probabilities have equal occupation probabilities at this point. This result is shown in Fig. (1e), where we quantify the degree of entanglement by using the concurrence. From this figure, we see that the degree of entanglement is increased as the time increased and reaches to the unity at $\tau \simeq 2.5$. For large time, the system will therefore reaches a state where all the occupation probabilities vanish. At specific time, $\tau \simeq 5$, the system turns into a product state, (see Fig. 1a). Also, we consider $\Delta = 1$ and all other parameters are fixed. As Δ, increased, the system turns into a product round $\tau = 5.4$. This means that the time of the entanglement survival is increased. This phenomenon is shown clearly by comparing Fig. 1a

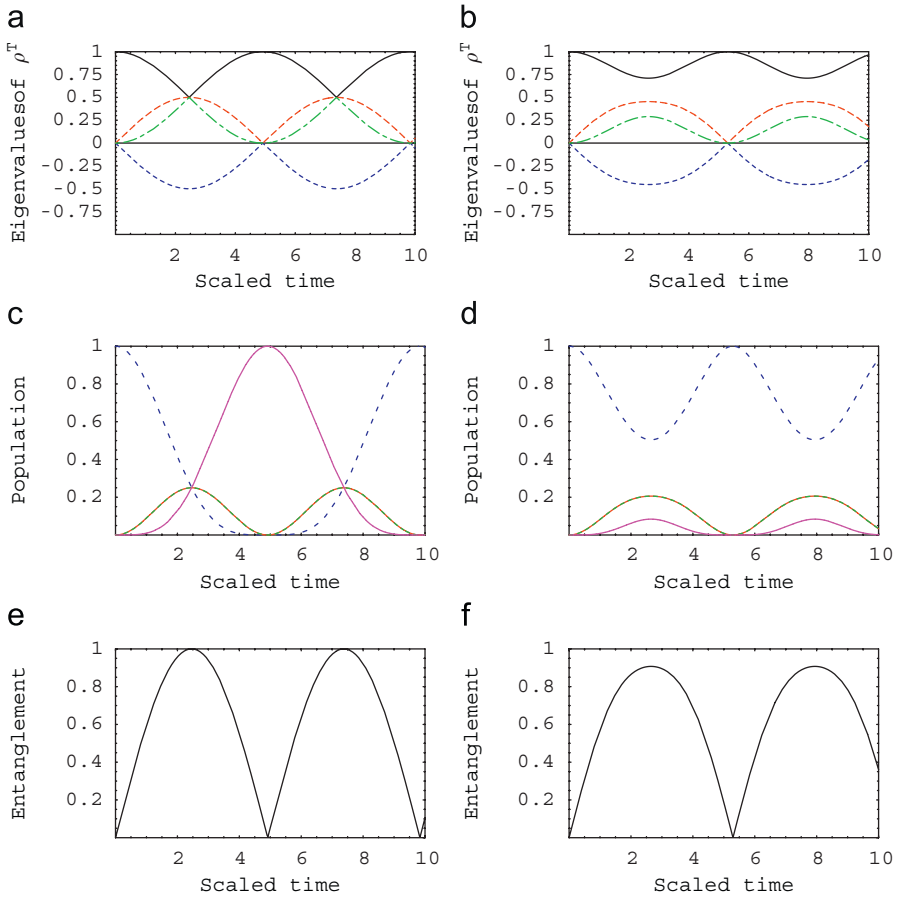

Fig. 1. The behavior of the eigenvalues of the partial transpose ρ^{T_2} is shown in (a, b). The populations are shown in (c, d) and the degree of entanglement is plotted in (e, f). We assume $n=1, C_{jg}=\frac{5}{2}$ and $\Delta=0$ for (a, c, e) while for (b, d, f), $\Delta=1$.

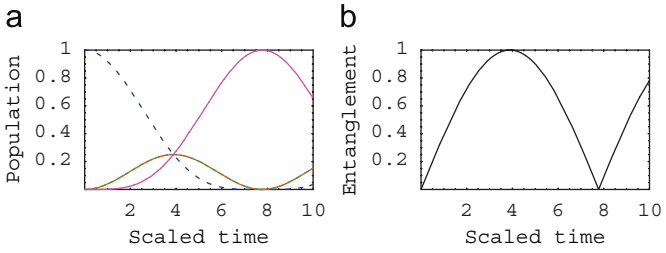

Fig. 2. The same as Fig. 1, but $\Delta=0, C_{jg}=\frac{2}{5}$.

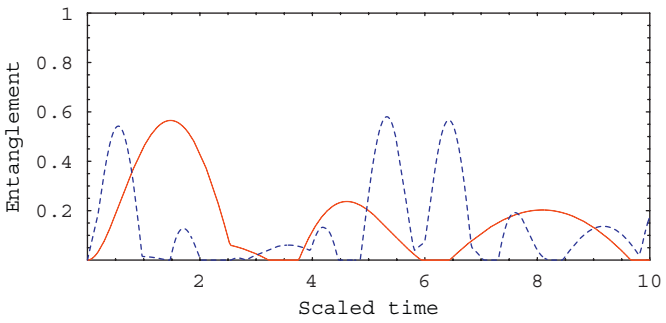

Fig. 3. The degree of entanglement for a system is initially prepared in the superposition state $|\psi_0\rangle = a|n,g\rangle + b|n,e\rangle$ with $a=1/\sqrt{2}$, $b=\sqrt{1-a^2}$ and $C_{jg}=\frac{5}{2}$. The solid and the dot curves for $\Delta=0$ and 1, respectively.

and b. The behavior of the occupation probability is seen in Fig. 1d, where there is no intersection point between the off diagonal occupation probabilities and the diagonal ones. So in this case one cannot get a maximum entangled state. As time goes on, $\tau \simeq 5.4$, the system becomes a separable, which agrees with that depicted in Fig. 1b. The amount of entanglement contained in the output state is shown in Fig. 1f. From this figure, one can notice that the maximum entanglement is less than unity. So by increasing the detuning parameter, one can increase the survival time of entanglement of the expense of the degree of entanglement.

The effect of the ratio of C_{jg} is seen in Fig. 2, where we consider a small value of this ratio. We investigate the behavior of the occupation probabilities and the degree of entanglement. As one decreases C_{jg}, the first maximum entangled state is obtained at $\tau=4$. This means that for small ratio, one takes a larger time to generate maximum entangled state. Also, the point at which the system turns into a separable state is shifted. The degree of entanglement contained in this state is shown in Fig. 2b, where it has a unity value at the maximum entangled state. So, the ratio C_{jg} has no effect on the degree of entanglement.

Secondly, we assume that the Cooper pair box is prepared in a superposition state. So the initial state of the system, $|\psi_0\rangle = a|n,g\rangle + b|n,e\rangle$. In Fig. 3, we plot the degree of entanglement where we assume that $a=\frac{1}{2}$, $C_{jg}=\frac{5}{2}$ and different values for the detuning parameter Δ. In this case, the effect is completely different (see Fig. 3a), where the degree of entanglement is small compared with that depicted in Fig. 1e. Also, the survival time of entanglement is small as compared with the previous case, where the system behaves as a separable system several times in a small range of time. In Fig. 3b, we increase the value of the detuning parameter ($\Delta=1$). It is clear that the instability of the system

increases and both the sudden death [27] and sudden birth of entanglement is seen [28]. Now, by controlling the Cooper pair box parameters and the field parameters, one can generate entangled state with high degree of entanglement. One of the best strategies is preparing the Cooper pair box in the excited or the ground state. If the charged qubit and the field at exact resonance i.e. $\Delta = 0$, one can generate a maximum entangled state. By decreasing the ratio C_{jg}, the survival time of entanglement becomes larger.

4. Teleportation

In what follows, we use the generated entangled state (4) to perform the quantum teleportation. Assume that Alice is given an unknown quantum state defined by

$$|\Psi\rangle = \lambda_1|0\rangle + \lambda_2|1\rangle, \tag{9}$$

where $\lambda_1^2 + \lambda_2^2 = 1$. The task of Alice is sending the state (9) to Bob through their quantum channel (4). To reach this aim, Alice and Bob use the original teleportation protocol [8]. The total state of the system is $\rho_\psi \otimes \rho^{out}$ where Alice makes measurements on the given qubit and her own qubit, then she sends her results through a classical channel to Bob. As soon as Bob receives the classical data, he performs a suitable unitary operation on his qubit to get the teleported state. Let us assume that Alice measures $|\psi^+\rangle$, then the density operator on Bob's hand is given by

$$\rho_{Bob} = \tfrac{1}{2}(|\lambda_1|^2|B_n|^2|0\rangle\langle 0| + \lambda_1\lambda_2^* B_n A_n^*|0\rangle\langle 1| + \lambda_2^*\lambda_1 A_n B_n^* + |\lambda_2|^2|A_n|^2|1\rangle\langle 1|) \tag{10}$$

with

$$A_n = \cos(\gamma\mu_n\tau) - \frac{i\Delta}{\mu_n}\sin(\gamma\mu_n\tau), \quad B_n = \frac{i\lambda\sqrt{n}}{\mu_n}\sin(\mu_n t), \tag{11}$$

where we assume that the Cooper pair box is prepared in the ground state and the field in the Fock state i.e. $|\psi_0\rangle = |g, n\rangle$. In Fig. 4, we plot the fidelity of the teleported state (10) for different values of the field and the Cooper pair box parameters. The effect of the detuning parameter is seen in Fig. 4a, where we assume that there is a one photon inside the cavity and the ratio $C_{jg} = 5$. From this figure, it is clear that as one increases Δ, the fidelity of the teleported state decreases. This is due to the fact that, for large values of the detuning parameter, the degree of entanglement decreases as it is clear from Fig. 1f. Since, the entangled time, the time in which the entanglement survives, increases for small values of the detuning parameter, the fidelity vanishes faster for large values of Δ. The effect of C_{jg} is shown in Fig. 4b, where for small values of this ratio, the fidelity reaches its maximum value faster than the large values of C_{jg}. On the other hand, for different values of this ratio there is no effect on the maximum values of the fidelity. Also, as one increases this ratio, the interval of time in

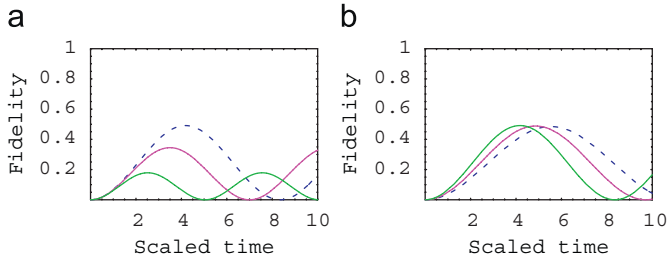

Fig. 4. The fidelity of the teleported state at Bob's hand (a) for $C_{jg} = 5$, number of photon, $n = 1$ and dot, solid and the dashed curves for $\Delta = 0.1, 0.5$ and 1, respectively. (b) The same as (a) but $\Delta = 0.1$ and the dot, the solid and the dash curves for $C_{jg} = \frac{5}{3}, \frac{5}{2}$ and 5, respectively.

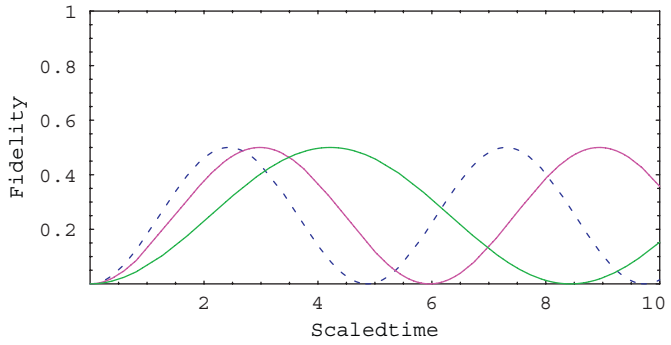

Fig. 5. The same as Fig. 4, with $C_j/C_g = 5$ and $\Delta = 0.1$ where the dot, the solid and the dash curves for $n = 3, 2, 1$, respectively.

which the channel is available for quantum teleportation increases.

In Fig. 5, we investigate the effect of different values of the number of photons inside the cavity. It is clear that for small values of n, the possibility of sending information with non-vanishing fidelity by using quantum teleportation increases. For larger values of n, the possibility of quick interaction increases and consequently one can gets an entangled state much faster.

5. Conclusion

We have employed the Cooper pair box to generate entangled state. We showed that it is possible to generate a maximum entangled state by controlling the capacities and the detuning parameter. Also, one can use the generated entangled state to perform a quantum teleportation. We have investigated the effect of the detuning parameter, the ratio of capacities and the photon number inside the cavity on the fidelity of the teleported state. For small values of the detuning parameter and the photon number one can teleportate the given state with larger fidelity. On the other hand, by increasing the ratio of the capacities, the intervals of time in which one can teleportate the state with non-vanishing fidelity are increased.

References

[1] D.V. Averin, Solid State Comm. 105 (1998) 659;
Y. Makhlin, G. Schon, A. Shnirman, Nature 398 (1999) 305.
[2] J. Mooij, T. Orlando, L. Levitov, L. Tian, C.H. van der Wal, S. Lloyd, Science 285 (1999) 1036.
[3] A. Blais, A. Zagoskin, Phys. Rev. A 61 (2000) 042308.
[4] D. Loss, D. DiVincenzo, Phys. Rev. A 57 (1998) 120.
[5] M. Zhang, J. Zpu, B. Shao, Internat. J. Modern Physica B 16 (2002) 4767;
W. Krech, T. Wanger, Phys. Lett. A 275 (2000) 159;
H.S. Ding, S.P. Zhao, G.H. Chen, Q.S. Yang, Physica C 382 (2002) 431.
[6] Yu. Makhlin, G. Schön, A. Shnirman, Nature 398 (1999) 305;
G. Schön, Yu. Makhlin, A. Shnirman, Physica C 352 (2001) 113;
J.Q. You, F. Nori, Physica E 18 (2003) 33.
[7] G. Wendin, Roy. Soc. London A 361 (2003) 1323;
A. Niskanen, J. Vartiainen, M. Salomaa, Phys. Rev. Lett. 90 (2003) 197901.
[8] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W. Wootters, Phys. Rev. Lett. 70 (1993) 1895.
[9] A. Cabello, Phys. Rev. Lett 89 (2003) 100402;
B. Zeng, X.S. Liu, S.Y. Li, G.L. Long, Comm. Theoret. Phys. 38 (2003) 537.
[10] H.-J. Cao, H.C. Zhong, S. He-Shan, Phys. Scripta 78 (2008) 015002.
[11] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eible, H. Weinfurter, A. Zeilinger, Nature 390 (1997) 575;
D. Boschi, S. Branca, F.D. Martini, L. Hardy, S. Popescu, Phys. Rev. Lett. 80 (1998) 1121.
[12] R. Migliore, A. Messina, A. Napoli, European Phys. J. B 13 (2000) 585;
R. Migliore, A. Messina, A. Napoli, European Phys. J. B 22 (2001) 111.
[13] Y. Makhlin, G. Schon, A. Shnirman, Rev. Modern Phys. 73 (2001) 357;
M. Abdel-Aty, H.F. Abdel-Hameed, N. Metwally, Physica C 452 (2007) 29.
[14] T.S. Tsai, Y. Nakamura, Yu. Pashkin, Physica C 367 (2002) 191;
Yu.A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. Averin, T. Tilma, F. Nori, J.S. Tsai, Physica C 426 (2005) 1552.
[15] J. You, F. Nori, Physica E 18 (2003) 33.

[16] W.P. Schleich, Quantum Optics in Phase Space, Wiley-VCH, Berlin, 2001;
S. Bose, I. Fuentes-Guridi, P.L. Knight, V. Vedral, Phys. Rev. Lett. 87 (2001) 050401;
S. Bose, I. Fuentes-Guridi, P.L. Knight, V. Vedral, Phys. Rev. Lett. 87 (2001) 279901(E);
S. Scheel, J. Eisert, P.L. Knight, M.B. Plenio, J. Modern Opt. 50 (2003) 881.
[17] M.C. Goorden, M. Thorwart, M. Grifoni, Phys. Rev. Lett. 93 (2004) 267005;
M. Thorwart, E. Paladino, M. Grifoni, Chem. Phys. 296 (2004) 333.
[18] M.C. Goorden, M. Thorwart, M. Grifoni, European Phys. J. B 45 (2005) 405.
[19] M. Abdel-Aty, J. Phys. A 41 (2008) 185304.
[20] A. Peres, Phys. Rev. Lett. A 77 (1996) 1413.
[21] M. Horodecki, P. Horodecki, R. Horodecki, Phys. Lett. A 222 (1996) 1.
[22] S. Hill, W.K. Wootters, Phys. Rev. Lett. 78 (1997) 5022.
[23] C.H. Bennett, H.J. Bernstein, S. Popescu, B. Schummacher, Phys. Rev. A 53 (1996) 2046.
[24] W.K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.
[25] M. Khasin, R. Kosloff, D. Steinitz, Phys. Rev. A 75 (2007) 052325;
D. Mundarain, J. Stephany, quant-ph 0712.1015, 2007.
[26] P. Horodecki, Phys. Lett 232 (1997) 333;
J. Eisert, M.B. Plenio, J. Modern Opt. 46 (1999) 145;
J. Lee, M.S. Kim, Y.J. Park, S. Lee, J. Modern Opt. 47 (2000) 2151.
[27] T. Yu, J.H. Eberly, Opt. Comm. 264 (2006) 393;
M. Yonac, T. Yu, J.H. Eberly, J. Phys. B 39 (2006) S621;
T. Yu, J.H. Eberly, J. Modern Opt. 54 (2007) 2289–2296.
[28] C.E. Lopez, G. Romero, F. Lastra, E. Solano, J.C. Retamal, Phys. Rev. Lett. 101 (2008) 080503.